Partie I :

Un échantillon de taille n = 1000 restaurants est tiré au hasard de toute la région. Sur cet échantillon, on observe 150 restaurants de statut illégal.

1)- Donner l'estimateur de **p**. Calculer son espérance et sa variance. Quelles sont ses propriétés?

2)- Donner l'intervalle de confiance de **p** à 95%.

Partie II :

En vue d'améliorer la précision de l'estimation précédente, on adopte un plan de sondage stratifié proportionnel (selon les sous-régions).

1)- Déterminer l'allocation d'un échantillon stratifié proportionnel de taille globale n = 1000 sur les différentes sous-régions.

2)- Cet échantillon donne dans les sous-régions les proportions observées suivantes :

Sous-région R_i	R_1	R_2	R_3	R_4
Proportion observée des restaurants de statut illégal	0.20	0.15	0.25	0.10

Donner l'intervalle de confiance de **p** à 95%.

2010/11/06

اللجنة الفاحصة

مجلس الخدمة المدنية
اللجنة الفاحصة
مباراة للتعيين في بعض المراكز الشاغرة وللتعاقد على بعض المهام لدى وزارة السياحة
لوظيفة إحصائي
مسابقة في الإحصاء الرياضي ونظرية العينات — المدة ساعتان

Problem I:

In a given region of Lebanon, a random sample of 100 households coming from abroad to spend August shows that 40 households have rented at least one car for their transportation.
Determine the optimal size for a sample that allows estimating the true proportion of households coming from abroad to spent August that have rented at least one car for their transportation with a precision of 2% and 95% of confidence..

Problem II:

The person in charge of the tourism office attached to a historic site decides to estimate the total amount spent by this site visitors on the « souvenirs » purchasing during a certain week-end.

1) – In the first day of the week-end and on a random sample of n = 50 visitors, we observe an average spending of 75000 L.L. with an observed standard deviation of 25000 L.L. Give a confidence interval (at 95%) for the true average amount spent by a visitor during his visit, this day, to the site.

2) - In the second day and on a random sample of n = 60 visitors, we observe an average spending of 90000 L.L. with an observed standard deviation of 30000 L.L.
Can we judge that (at risk of 5%) the average amount spent by a visitor on the « souvenirs » purchasing has not varied from the first day to the second day?

3) – Given that the number of visitors recorded during the given week-end was respectively 1000 visitors for the first day and 1500 for the second day:
a) – Estimate the total amount spent by these visitors on « souvenirs » purchasing.
b) – Estimate this amount by a confidence interval at 95%.

Problem III:
In a certain country, the Statistics Department in the Ministry of Tourism has the mission to survey the legal status of restaurants operating in a given region. This region is portioned into four sub-regions denoted by R_1, R_2, R_3 et R_4. A recent census shows that the total number of restaurants (N) operating in these sub-regions is distributed as follows:

Sub-region R_i	R_1	R_2	R_3	R_4	The entire region
Number of operating restaurants N_i	1500	2000	3000	3500	10000

The target of this survey is estimating p = the proportion of restaurants of illegal status and operating in the region of interest..

Turn over

Part I :

A sample of size n = 1000 restaurants is selected at random from the entire region. On this sample, we observe that 150 restaurants have illegal status.

1) - Give the estimator of **p**. Compute its expectation as well as its variance. What are its properties?

2) - Give the confidence interval of **p** at 95%.

Part II :

In order to improve the precision of the previous estimate, we adopt a Proportionate Stratified Sampling Design according to sub-regions.

1) – Determine the allocation of a proportionate stratified sample having a global size n=1000 on the different sub-regions.

2) – This sample gives in the different sub-regions the following observed proportions:

Sub-region R_i	R_1	R_2	R_3	R_4
Observed proportion of restaurant with illegal status	0.20	0.15	0.25	0.10

Give the confidence interval of **p** at 95%.

2010/11/06

اللجنة الفاحصة

$N(0,1)$ Table de Gauss

z	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0,0	0,5000	0,5040	0,5080	0,5120	0,5160	0,5199	0,5239	0,5279	0,5319	0,5359
0,1	0,5398	0,5438	0,5478	0,5517	0,5557	0,5596	0,5636	0,5675	0,5714	0,5753
0,2	0,5793	0,5832	0,5871	0,5910	0,5948	0,5987	0,6026	0,6064	0,6103	0,6141
0,3	0,6179	0,6217	0,6255	0,6293	0,6331	0,6368	0,6406	0,6443	0,6480	0,6517
0,4	0,6554	0,6591	0,6628	0,6664	0,6700	0,6736	0,6772	0,6808	0,6844	0,6879
0,5	0,6915	0,6950	0,6985	0,7019	0,7054	0,7088	0,7123	0,7157	0,7190	0,7224
0,6	0,7257	0,7291	0,7324	0,7357	0,7389	0,7422	0,7454	0,7486	0,7517	0,7549
0,7	0,7580	0,7611	0,7642	0,7673	0,7704	0,7734	0,7764	0,7794	0,7823	0,7852
0,8	0,7881	0,7910	0,7939	0,7967	0,7995	0,8023	0,8051	0,8078	0,8106	0,8133
0,9	0,8159	0,8186	0,8212	0,8238	0,8264	0,8289	0,8315	0,8340	0,8365	0,8389
1,0	0,8413	0,8438	0,8461	0,8485	0,8508	0,8531	0,8554	0,8577	0,8599	0,8621
1,1	0,8643	0,8665	0,8686	0,8708	0,8729	0,8749	0,8770	0,8790	0,8810	0,8830
1,2	0,8849	0,8869	0,8888	0,8907	0,8925	0,8944	0,8962	0,8980	0,8997	0,9015
1,3	0,9032	0,9049	0,9066	0,9082	0,9099	0,9115	0,9131	0,9147	0,9162	0,9177
1,4	0,9192	0,9207	0,9222	0,9236	0,9251	0,9265	0,9279	0,9292	0,9306	0,9319
1,5	0,9332	0,9345	0,9357	0,9370	0,9382	0,9394	0,9406	0,9418	0,9429	0,9441
1,6	0,9452	0,9463	0,9474	0,9484	0,9495	0,9505	0,9515	0,9525	0,9535	0,9545
1,7	0,9554	0,9564	0,9573	0,9582	0,9591	0,9599	0,9608	0,9616	0,9625	0,9633
1,8	0,9641	0,9649	0,9656	0,9664	0,9671	0,9678	0,9686	0,9693	0,9699	0,9706
1,9	0,9713	0,9719	0,9726	0,9732	0,9738	0,9744	0,9750	0,9756	0,9761	0,9767
2,0	0,9772	0,9778	0,9783	0,9788	0,9793	0,9798	0,9803	0,9808	0,9812	0,9817
2,1	0,9821	0,9826	0,9830	0,9834	0,9838	0,9842	0,9846	0,9850	0,9854	0,9857
2,2	0,9861	0,9864	0,9868	0,9871	0,9875	0,9878	0,9881	0,9884	0,9887	0,9890
2,3	0,9893	0,9896	0,9898	0,9901	0,9904	0,9906	0,9909	0,9911	0,9913	0,9916
2,4	0,9918	0,9920	0,9922	0,9925	0,9927	0,9929	0,9931	0,9932	0,9934	0,9936
2,5	0,9938	0,9940	0,9941	0,9943	0,9945	0,9946	0,9948	0,9949	0,9951	0,9952
2,6	0,9953	0,9955	0,9956	0,9957	0,9959	0,9960	0,9961	0,9962	0,9963	0,9964
2,7	0,9965	0,9966	0,9967	0,9968	0,9969	0,9970	0,9971	0,9972	0,9973	0,9974
2,8	0,9974	0,9975	0,9976	0,9977	0,9977	0,9978	0,9979	0,9979	0,9980	0,9981
2,9	0,9981	0,9982	0,9982	0,9983	0,9984	0,9984	0,9985	0,9985	0,9986	0,9986
3,0	0,9987	0,9987	0,9987	0,9988	0,9988	0,9989	0,9989	0,9989	0,9990	0,9990

La table de Gauss donne les probabilités $\Phi(z)$ pour des valeurs positives de z telles que :

$$P\{Z \leqslant z\} = \Phi(z) = A$$

Note : $\Phi(-z) = 1 - \Phi(z)$